gion, the discrepancy between relativistic and non-relativistic ph excitation is very small both for PV and for PSπ NN coupling, so people is save to use non-relattivistic approximation. For high energy momentum transfer, howeveer, the N$\bar{N}$ excitation will have some ontributions also. But quantitative calulation depends on the scheme of renormalization and explicitly speaking the PV π NN coupling is not renormalizable. The pion condensation is not like to happen due to Landau damping. This is just a preliminary conclusion since N$\bar{N}$ excitation, short range correlation, △ -hole excitation are not included yet.

Another important conclusion is that people can not get the correct formulae for ph excitation when propagators G_F, G_D are used to calculate the polarization insertion if $\Pi_F^{PS(PV)}$ is neglected.

The author Liu is very grateful to Prof. Ma Weixing and Prof. Jiang Huanqing for valuable discussions.

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相对论 Lindhard函数和粒子 空穴激发

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摘 要 用两种方法计算子核物质中π介子传播了的极化插入.通过计算π介子的色散关系,发现在低动量转移区相对论与非相对论的差别很小;π NN的赝标量与赝矢量耦合之间的差别即使对大动量转移区也很小;在所有情况下,没有π凝聚现象出现.

关键词 相对论粒子 空穴激发, Lindhard函数,π 介子色散关系

分类号 O 571. 2, O 572. 339, O 413. 3

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中山大学学报 (自然科学版)
第 35卷 第 5期 ACTA SCIENTIARUM NATURALIUM Vol. 35 No. 5
1996年 9月 UNIVERSITATIS SUNYATSENI Sep. 1996

RELATIVISTIC LINDHARD FUNCTION FOR PARTICLE-HOLE EXCITATION*

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Abstract The nucleon 1-loop polarization insertion for pion propagator in nuclear matter is calculated in two ways. One way is that the nucleon propagator is split into Feynman and density dependent nucleon propagator(G_F and G_D) and another way is to use particle-hole-antiparticle representation (S_p, S_h, $S_{\bar{p}}$) of the nucleon propagator. The polarization insertion can be expressed in terms of Relativistic Lindhard functions defined in the text. By calculating the dispersion relation it is found that: first, the difference between relativistic and non-relativistic approximation is small in the low energy-momentum transfer region. Second, the difference between π NN pseudo-scalar (PS) and pseudo-vector (PV) coupling is small also even for large energy-momentum transfer. Third, in all cases the zero sound braneh of the dispersion relation is completely damped so there is no pion condensation can happen in this calculation.

Keywords relativistic particle-hole excitation, Lindhard function, π dispersion relation

Classification number O571. 2, O572. 339, O413. 3

1 Introduction

The pion propagator or the polarization insertion of the pion propagator has been studied extensively in the non-relativistic approximations[1]. Besides the particle-hole(ph) excitation, △-hole (△h) excitation and NN short range correlation are all included[2]. These mechanisms are really work very well in the low energy-momentum region. But one can ask a basic question: to what energy scale the non-relativistic formalism still holds valid, or put it in another word, what is the discrepancy between non-relativistic and relativistic formalism in the same order of approximation.

Usually, the polarization insertion for e, k[3] and π[4] propagators were calculated in the Walecka's model in the following way: the nucleon propagator in nuclear matter can be split into Feynman part (G_F) and density dependent part (G_D). For example the polarization insertion for π NN PS coupling case can be split into Feynman part Π_F^{PS} and density dependent part Π_F^{PS}, and they can be expressed as follows:

$$\Pi_F^{PS}(q) = -\frac{g_{\pi NN}^2}{(2\pi)^3}\int \frac{d\boldsymbol{P}}{E_P E_{P-q}}\left[4E_{p-q} + q^2\left(\frac{1}{E_p + E_{p-q} - q_0 - i\epsilon} + \frac{1}{E_p + E_{p-q} + q_0 - i\epsilon}\right)\right] \qquad (1)$$

* 国家自然科学基金资助项目
收稿日期: 1995-11-15 刘良钢,男,36岁,教授

$$\Pi_D^{PS}(q)=\frac{g_{\pi NN}^2}{(2\pi)^3}\int\frac{d\boldsymbol{p}}{E_p E_{p-q}}\{(1-n_p)n_{p-q}\cdot q^2(\frac{1}{E_p-E_{p-q}-q_0-i\varepsilon}+\frac{1}{E_p-E_{p-q}+q_0-i\varepsilon})+$$
$$n_p[4E_{p-q}+q^2(\frac{1}{E_p+E_{p-q}+q_0-i\varepsilon}+\frac{1}{E_p+E_{p-q}-q_0-i\varepsilon})]\} \quad (2)$$

where $g_{\pi NN}$ is π NN PS coupling constant. $E_p=\sqrt{\boldsymbol{p}^2+\tilde{m}^2}$, $\tilde{m}$ is nucleon effective mass. n_p is the nucleon distribution function. It is obvious that Π_F^{PS} is divergent and its form depends on the scheme of renormalization. usually people negleted its contribution and only remains density dependent part. The first term in eq. (2) looks like a ph excitation (later we can understand that this is not complete) but the second term violates the Pauli principle simply because the excited nucleon is in the Fermi sea. In this sense, we can not know what is the ph and particle-antiparticle ($N\bar{N}$) contributions, unless we start from the particle-hole-antiparticle representation of the nucleon propagator.

2 Relativistic and Nonrelativistic Lindhard Function

The nucleon propagator in nuclear matter can also be expressed in the following way:

$$G(p)=S_p(p)+S_h(p)+S_{\bar{p}}(p) \quad (3)$$
$$S_p(p)=(\tilde{m}/E_p)[(1-n_p)\Lambda_+(\boldsymbol{p})]/(p_0-E_p+i\varepsilon) \quad (4)$$
$$S_h(p)=(\tilde{m}/E_p)[(n_p\Lambda_+(\boldsymbol{p})/(p_0-E_p-i\varepsilon)] \quad (5)$$
$$S_{\bar{p}}(p)=(\tilde{m}/E_p)[\Lambda_-(-\boldsymbol{p})/(p_0+E_p-i\varepsilon)] \quad (6)$$

here S_p, S_h, $S_{\bar{p}}$ is the particle, hole, antiparticle propagator and $\Lambda_+(\boldsymbol{p})$, $\Lambda_-(\boldsymbol{p})$ are the particle and antiparticle projection operator, respectively[5],

$$\Lambda_+(\boldsymbol{p})=(\gamma\cdot p+\tilde{m})/2\tilde{m} \quad (7)$$
$$\Lambda_-(\boldsymbol{p})=(-\gamma\cdot p+\tilde{m})/2\tilde{m} \quad (8)$$

The polarization insertion for pion propagator can be recalculated by employing these propagators. It is nothing but just the ph and $N\bar{N}$ excitations. First let us define relativistic Lindhard functions $U(q)$, $V(q)$ as follows:

$$U(q)=-\frac{4}{(2\pi)^3}\int\frac{d\boldsymbol{p}}{E_pE_{p-q}}(1-n_p)n_{p-q}\cdot(\frac{1}{E_p-E_{p-q}-q_0-i\varepsilon}+\frac{1}{E_p-E_{p-q}+q_0-i\varepsilon}) \quad (9)$$

$$V(q)=-\frac{4}{(2\pi)^3}\int\frac{d\boldsymbol{p}}{E_pE_p}(1-n_{p-q})\cdot(\frac{1}{E_p+E_{p-q}-q_0-i\varepsilon}+\frac{1}{E_p+E_{p-q}+q_0-i\varepsilon}) \quad (10)$$

In the limit $\boldsymbol{p}\to 0$,

$$U(q)\to(1/\tilde{m}^2)U_N(q)+O(|\boldsymbol{q}|/\tilde{m}) \quad (11)$$

where $U_N(q)$ is the well known non-relativistic Lindhard function[2,6], and its real part can be written as follows:

$$\mathrm{Re}U_N(\nu,q_F)=\frac{\tilde{m}k_F}{\pi^2}\{-1+\frac{1}{2q_F}[1-(\frac{\nu}{q_F}-\frac{q_F}{2})^2]\ln|(\frac{\nu}{q_F}-\frac{q_F}{2}+1)/(\frac{\nu}{q_F}-\frac{q_F}{2}-1)|$$
$$-\frac{1}{2q_F}[1-(\frac{\nu}{q_F}+\frac{q_F}{2})^2]\ln|(\frac{\nu}{q_F}+\frac{q_F}{2}+1)/(\frac{\nu}{q_F}+\frac{q_F}{2}-1)|\} \quad (12)$$

here $\nu=q_0\tilde{m}/k_F^2$, $q_F=|q|/k_F$, k_F is Fermi momentum.

The real part of $V(q)$ is divergent thus there is no non-relativistic correspondence. The real part of $U(q)$ is easy to evaluate and its imaginary part verified agrees with that of ref. (4).

In Fig. 1 we show a comparison between $m^2U(q)$ and $U_N(q)$ for two typical value of

q_0, one is $q_0 = 0.15k_F$ corresponds to the curves of higher peak and $q_0 = 0.4941\,k_F$ is for another. We choose $\tilde{m} = m = 938.5$ MeV, nucleon density $\rho_0 = 0.2\ \mathrm{fm}^{-3}$. The y-axis is in unit of k_F^{-2}. The discrepancy between them will be due to relativistic effects. We can see that for both cases the difference becomes larger when momentum goes larger. That is what we expected.

Fig. 1 Relativistic (dotted line) and non-relativistic (dashed line) Lindhard function versus momentum q_F for fixed frequency q_0. For explanations see text

The pion polarization insertion can be expressed in terms of functions $U(q)$, $V(q)$ as follows:

$$\Pi^{PS(PV)}(q) = \Pi_{ph}^{PS(PV)}(q) + \Pi_{N\bar{N}}^{PS(PV)}(q) \tag{13}$$

$$\Pi_{ph}^{PS}(q) = \frac{2g_{\pi NN}^2}{(2\pi)^3}\int \frac{\mathrm{d}\boldsymbol{p}}{E_p E_{p-q}}(1-n_p)n_{p-q}(E_p - E_{p-q}) - \frac{1}{4}g_{\pi NN}^2 U(q) \tag{14}$$

$$\Pi_{ph}^{PV}(q) = \frac{2f_{\pi NN}^2}{(2\pi)^3 m_\pi^2}\int \frac{\mathrm{d}\boldsymbol{p}}{E_p E_{p-q}}(1-n_p)n_{p-q}(E_{p-q} - E_p)\cdot$$

$$[(E_p + E_{p-q})^2 - q^2] - \frac{f_{\pi NN}^2}{m_\pi^2}\tilde{m}^2 q^2 U(q) \tag{15}$$

$$\Pi_{N\bar{N}}^{PS}(q) = \frac{2g_{\pi NN}^2}{(2\pi)^3}\int \frac{\mathrm{d}\boldsymbol{p}}{E_p E_{p-q}}(1-n_p)(E_p + E_{p-q}) + \frac{1}{4}g_{\pi NN}^2 V(\boldsymbol{p}) \tag{16}$$

$$\Pi_{N\bar{N}}^{PV}(q) = \frac{2f_{\pi NN}^2}{(2\pi)^3 m_\pi^2}\int \frac{\mathrm{d}\boldsymbol{p}}{E_p E_{p-q}}(1-n_p)(E_p + E_{p-q})\cdot$$

$$[(E_p - E_{p-q})^2 - \boldsymbol{q}^2] + \frac{f_{\pi NN}^2}{m_\pi^2}\tilde{m}^2 q^2 V(\boldsymbol{q}) \tag{17}$$

It is easy to verify that $\Pi^{PS(PV)}(q) = \Pi_F^{PS(PV)}(q) + \Pi_D^{PS(PV)}(q)$. Compare Π_{ph}^{PS} with the first term in eq. (2), then we can understand that one can not get the correct ph excitation contribution by $\Pi_D^{PS}(q)$. The same is true for PV coupling.

The pionic ph excitation dispersion relation is calculated by following eqs.

$$\omega^2 - \boldsymbol{q}^2 - m_\pi^2 - \mathrm{Re}\,\Pi_{ph}^{PS(PV)}(\omega, |\boldsymbol{q}|) = 0 \tag{18}$$

in the relativistic case and

$$\omega^2 - \boldsymbol{q}^2 - m_\pi^2 - (f_{\pi NN}^2/m_\pi^2)\boldsymbol{q}^2\,\mathrm{Re}\,U_N(\omega, |\boldsymbol{q}|) = 0 \tag{19}$$

in the non-relativistic case.

3 Numerical Results

In Fig. 2, we show the relativistic and non-relativistic dispersion relation for PV cou

Fig. 2 Relativistic (dashed line) and non-relativistic (dotted line) dispersion relation for PV πNN coupling. In the hatched region indicated the imaginary part of the polarization insertion does not vanish

pling. k is in unit of k_F. $f_{\pi NN}/m_\pi$ is replaced by $g_{\pi NN}/2m$ and $g_{\pi NN}$ is chosen to be 13 in the numerical calculation. m_π = 140 MeV, ρ_0 = 0.2 fm^{-3}. We can see that for meson branch the difference increases when k and $|\boldsymbol{q}|$ increase. But the dashed line enters into space-like region at about q_F = 2.9. The zero sound branch almost overlaps each other for low q_F and has littlee difference for q_F larger than 1.4. The pion condensation is completely damped by ph decaysso it will not be stable.

In order to compare PS and PV couplings, their ph excitation dispersion relatiolns are calculated by using the same parameters as above and the results are shown in Fig. 3. Again we an see the difference is quite small for all k and q_F except that the meson branch of the PV coupling enters into space-like region but PS coupling doesn't.

Fig. 3 Relativistic dispersion relation for PS (dotted line) and PV (dashed line) πNN couplings. The parameters used are the same as those of Fig. 2

4 Summary and Conclusions

In conclusion from this study we note that in the low energy momentum transfer re-